NATIONAL HIGHWAY TRAFFIC SAFETY ADMINISTRATION

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REFINEMENT OF FINITE ELEMENT ANALYSIS OF AUTOMOBILE STRUCTURES UNDER CRASH LOADING

Volume 1: Summary Report

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Contract No. DOT HS-6-01364
Contract Amt. $86,660

October 1977
FINAL REPORT

Prepared For
U.S. DEPARTMENT OF TRANSPORTATION
National Highway Traffic Safety Administration
Washington, D.C. 20590

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No. DOT HS 803 465	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle REFINEMENT OF FINITE ELEMENT ANALYSIS OF AUTOMOBILE STRUCTURES UNDER CRASH LOADING VOLUME I		5. Report Date October 1977
		6. Performing Organization Code J6384
7. Author(s) Kin S. Yeung, R. E. Welch		8. Performing Organization Report No.
9. Performing Organization Name and Address IIT Research Institute 10 West 35th Street Chicago, Illinois 60616		10. Work Unit No.
		11. Contract or Grant No. DOT-HS-6-01364
12. Sponsoring Agency Name and Address Department of Transportation National Highway Traffic Safety Administration 2100 Second Street, S.W. Washington, D.C. 20590		13. Type of Report and Period Covered Summary Final Report 6/21/76 - 11/1/77
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

/A finite element computer program for use in the static and dynamic analyses of vehicle structure, including sheet metal, in a crash environment was developed in this research project. The developed computer program consists of the following features:

- Large displacement, nonlinear static and dynamic, and elastic and plastic including strain-rate effect
- With plate, three-dimensional beam and spring elements, and rigid links and a variety of three-dimensional beam end conditions
- Options of using either the explicit or implicit time integration procedures
- Options of specifying stress, mass and center of gravity, and energy output.

17. Key Words finite element computer program static and dynamic analyses vehicles crashworthiness	18. Distribution Statement Document is available to the public through the National Technical Information Service, Springfield, Virginia 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	21	

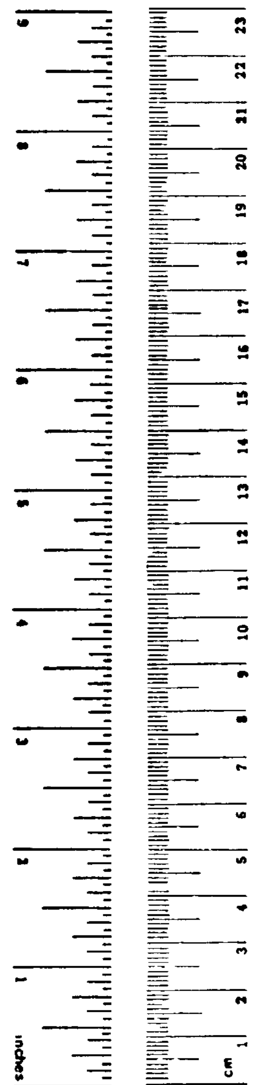
Form DOT F 1700.7 (8-69)

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

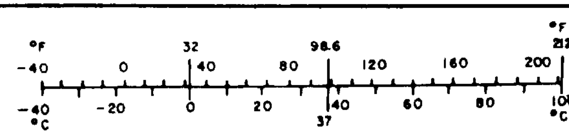
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10 000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



PREFACE

This final report entitled "Refinement of Finite Element Analysis of Automobile Structures Under Crash Loading", Summary Final Report Volume I, presents the results of a research project undertaken from 21 June 1976 to 1 November 1977 by IIT Research Institute (IITRI) for the Department of Transportation, National Highway Traffic Safety Administration (NHTSA) under Contract DOT-HS-6-01364 (IITRI Project J6384). The report is presented in two volumes. Volume I, Summary Final Report, and Volume II, Technical Final Report.

Mr. Tom Hollowell served as the NHTSA Contract Technical Manager. The IITRI Project Manager, initially was Dr. R. E. Welch (from June 1976 to December 1976) and subsequently Dr. K. S. Yeung, who is under general supervision of Mr. A. Longinow, Manager, Structural Analysis Section, and Dr. K. E. McKee, Director, Engineering Research Division. Mr. B. L. Kruchoski of IITRI made significant technical contributions to this report. Dr. R. L. Chiapetta incorporated the explicit moment curvature relationship and the strain rate effect.

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1. INTRODUCTION

The increased concern in recent years over vehicle safety has focused, in part, on the ability of a vehicle to sustain a crash event; to absorb, redirect and otherwise manage the severe forces and energy associated with the event; and thereby, to lessen the severity of the environment to which passengers are exposed. In addition to basic vehicle crashworthiness, serious attention has also been devoted to such factors as the propensity of a vehicle to damage in minor crash events and the threat posed to pedestrians or other vehicles by particular vehicle designs.

Since quantitative knowledge of the deformation characteristics of vehicle structures is an intrinsic requirement in all considerations of this type, it is not surprising that the developing concern for vehicle safety has given rise to intense and continuing efforts to provide mathematical descriptions of structural deformations in the crash environment. In this report the results are presented of a research project undertaken by IIT Research Institute (IITRI) for the Department of Transportation, to improve the existing WRECKER finite element computer program,* hereafter referred to as WRECKER I. This program is developed for use in the dynamic analysis of vehicle structure, including sheet metal, in a crash environment. The resultant computer program WRECKER II containing all of the analytical features present in WRECKER I, employs an explicit as well as implicit method solution. Additional features being undertaken in connection with WRECKER II involve improvements in the program input and output, a more versatile specification of boundary conditions, the development of generalized loading descriptions and special beam end conditions, and improvements in the material representation (strain rate effects and explicit moment curvature relations).

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* Welch, R.E., et al, "Finite Element Analysis of Automotive Structures Under Crash Loadings", DOT/NHTSA, Contract DOT-HS-105-3-697, IITRI Project J6321, May 1975.

2. ANALYSIS REQUIREMENTS

The response of vehicle structures under crash loadings is a complex process primarily involving:

- transient, dynamic behavior
- complicated framework and shell assemblages
- large deflections and rotations
- extensive plastic deformation
- static or quasi-static response.

Previous attempts at a formal analysis of this process have been only partly successful due to a variety of limitations which, in particular instances, have included inadequacies in element formulations, material representations or solution procedures. The work presented in this report represents an attempt to develop a finite element program which is specifically tailored to the class of problems inherent in vehicle crash response, and which employs or extends current avenues in finite element analysis which seem best suited to such problems. The field of nonlinear finite element analysis is currently an extremely active area of research with an extensive, related literature and a variety of methods and approaches. Consequently, a formal review of the field as background for the present analysis approach is not attempted. Instead, major features of the present work are briefly described, and some rationale is offered for their use in the context of vehicle analysis.

2.1 Coordinate Systems

Three different coordinate systems are used in describing the configuration of the model in space. They are global, element and nodal coordinate systems; see Figures 1 and 2.

The global coordinate system (x,y,z) is fixed in space and serves as an inertial frame of reference for the three translation motions.

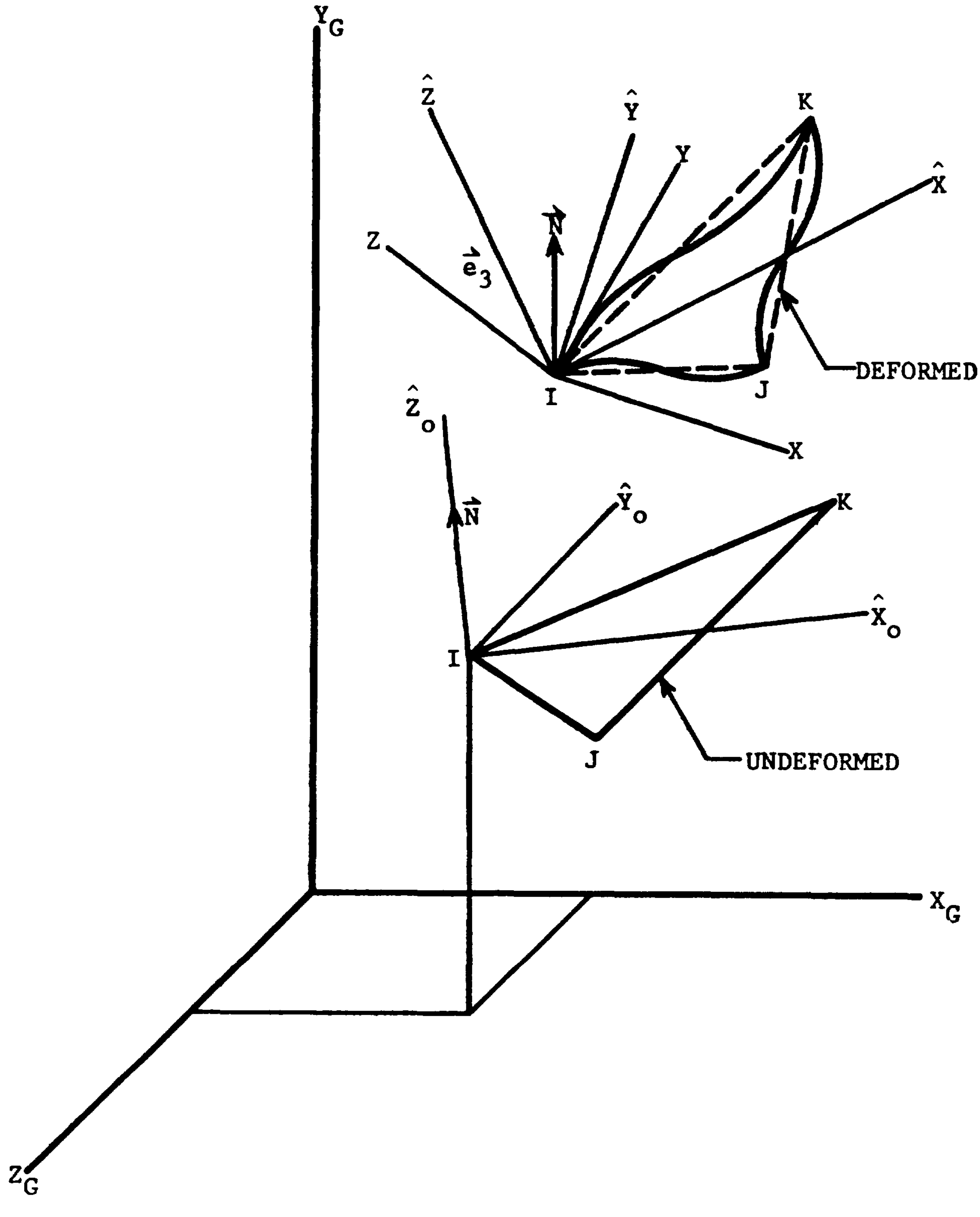

FIGURE 1. THREE-DIMENSIONAL PLATE ELEMENT.

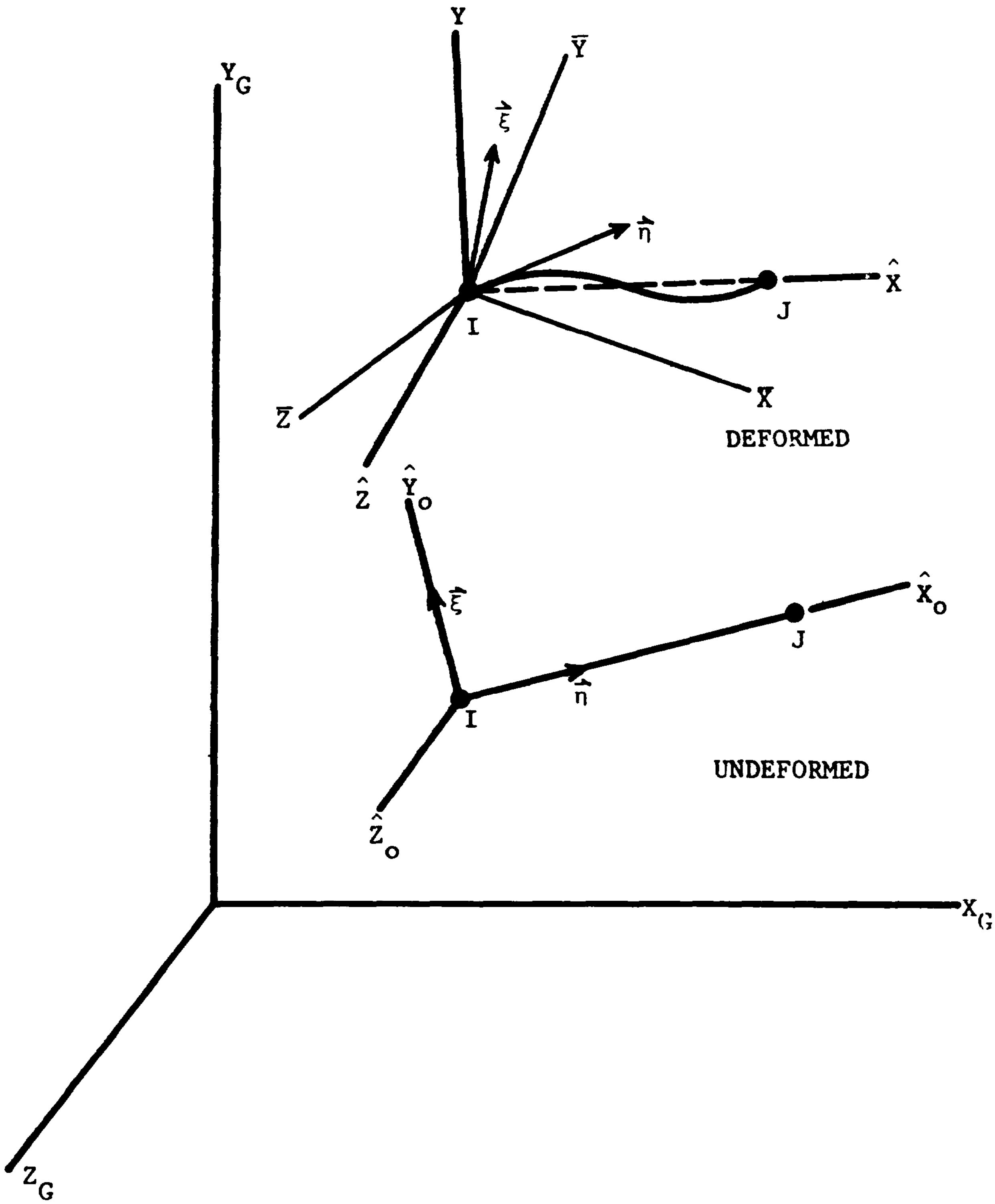

FIGURE 2. THREE-DIMENSIONAL BEAM ELEMENT.

The element coordinate system $(\hat{x},\hat{y},\hat{z})$ is embedded in each element and serves to define the rigid body rotation of the element as a reference for element distortions and forces. The components of a vector, $\vec{V}$, transform from the element coordinate system to the global coordinate system by the time dependent transformation

$$[V_G] = [E][\hat{V}] \tag{1}$$

where the elements of E coorespond columnwise with the global components of unit vectors $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ oriented with the element axes respectively. For triangular plate elements the coordinate system is defined by the normal to the plane formed by the three corner nodes and by a line bisecting the angle at a selected node. The remaining axis is determined by a plane formed by the normal and the angle bisector (Figure 1). For beams the element coordinate axes are defined by a line connecting the end points and by a plane through this line which bisects the defined positions of vectors coinciding with principal axes of the cross section at the ends of the beam (Figure 2). For springs only one element axis is required and is defined by a line joining the end points of the spring.

The nodal axes $(\bar{x},\bar{y},\bar{z})$ are attached to each node of an element. They initially coincide with the global axes and subsequently rotate with the node. The orientation of the nodal axes $(\bar{x},\bar{y},\bar{z})$ with respect to the global axes at any time during the motion is established by the components of three unit vectors $\vec{b}_1$, $\vec{b}_2$, $\vec{b}_3$ which remain fixed along the nodal axes $(\bar{x},\bar{y},\bar{z})$, respectively, as the node rotates. If the elements of B correspond columnwise with these three unit vectors then the components of any vectors, $\vec{V}$, transform from the nodal to global system by

$$[V_G] = [B][\bar{V}] \tag{2}$$

2.2 Solution Procedures

Two solution procedures, explicit and implicit are used for the time integration of the equations of motion. They are outlined below.

2.2.1 Explicit Time Integration - The choice of the explicit time integration procedure results in a program with minimum computer storage requirement and by far is the most efficient method. But the time step used must be quite small because of numerical stability if the structure being analyzed has a high frequency content.

The equations of motion are derivable from the principle of virtual work with the inertial forces treated as external forces. These equations of motion at a typical node point for translations and rotations, respectively, are then written as follows.

Translation

$$[M][\ddot{u}_G] = [f_G^E] - [f_G^I] \tag{3}$$

where

- $[M]$ the diagonal, lumped mass matrix for the node
- $[\ddot{u}_G] = [\ddot{u}_{xG}, \ddot{u}_{yG}, \ddot{u}_{zG}]$ is the acceleration vector for the node referred to a global system (X_G, Y_G, Z_G) common to all nodes
- $[f_G^E]$ the vector of external, applied loads at the node referred to the global system
- $[f_G^I]$ the vector of forces at the node in the global system which results from deformations in all elements associated with the node.

Rotation

$$\begin{bmatrix} \bar{I}_{xx} & -\bar{I}_{xy} & -\bar{I}_{xz} \\ -\bar{I}_{xy} & \bar{I}_{yy} & -\bar{I}_{yz} \\ -\bar{I}_{xz} & -\bar{I}_{yz} & \bar{I}_{zz} \end{bmatrix} \begin{bmatrix} \bar{\alpha}_x \\ \bar{\alpha}_y \\ \bar{\alpha}_z \end{bmatrix} = \bar{F}_E(t) - \bar{F}_I(x) - \bar{F}_\omega \qquad \begin{matrix} (4a) \\ (4b) \\ (4c) \end{matrix}$$

where

$$\bar{F}_\omega = \begin{bmatrix} \bar{F}_{\omega x} \\ \bar{F}_{\omega y} \\ \bar{F}_{\omega z} \end{bmatrix} = \begin{bmatrix} \bar{\omega}_y\bar{\omega}_z(\bar{I}_{zz}-\bar{I}_{yy})+\bar{\omega}_z\bar{\omega}_x\bar{I}_{xy}-\bar{\omega}_y\bar{\omega}_x\bar{I}_{xz}-(\bar{\omega}_y^2-\bar{\omega}_z^2)\bar{I}_{yz} \\ \bar{\omega}_z\bar{\omega}_y(\bar{I}_{xx}-\bar{I}_{zz})+\bar{\omega}_x\bar{\omega}_y\bar{I}_{yz}-\bar{\omega}_x\bar{\omega}_y\bar{I}_{yx}-(\bar{\omega}_z^2-\bar{\omega}_x^2)\bar{I}_{zx} \\ \bar{\omega}_x\bar{\omega}_y(\bar{I}_{yy}-\bar{I}_{xx})+\bar{\omega}_y\bar{\omega}_z\bar{I}_{zx}-\bar{\omega}_x\bar{\omega}_z\bar{I}_{zy}-(\bar{\omega}_x^2-\bar{\omega}_y^2)\bar{I}_{xy} \end{bmatrix} \begin{matrix} (4d) \\ (4e) \\ (4f) \end{matrix}$$

where all components are referred to a coordinate system $[\bar{x},\bar{y},\bar{z}]$ which initially coincides with the global axes and which subsequently rotates with the node (i.e., a set of rigid body axes for each node) and where $[\bar{I}_x,\bar{I}_y,\bar{I}_z,\bar{I}_{xy},\bar{I}_{yz}]$ are the mass moments of inertia, $[\overline{\alpha_x},\overline{\alpha_y},\overline{\alpha_z}]$ are instantaneous angular accelerations, $[\overline{\omega_x},\overline{\omega_y},\overline{\omega_z}]$ are the angular velocities, and where $[\bar{F}_E]$ and $[\bar{F}_I]$ are the external and internal moment vectors at the node point. The mass moments of inertia should be updated appropriately.

The dynamical uncoupled translational and rotational motions are based upon the assumption that the center of mass is chosen as the reference point.

The explicit temporal integration procedure employs the Newmark-Beta method. This method relates displacement, velocity, and acceleration at the beginning (u_o,v_o,a_o) and end (u_1,v_1,a_1) of a time interval, h, by the relations

$$u_1 = u_o + v_o h + (\tfrac{1}{2} - \beta)\ a_o h^2 + \beta h^2 a_1 \qquad (5a)$$

$$v_1 = v_o + \tfrac{h}{2}\ (a_o + a_1) \qquad (5b)$$

Solutions to the equations of motion, equations (3) and (4), may be obtained by using equations (5), (3), and (4) predicting u_1 and v_1 by means of equation (5) and correcting a_1 by means of equations (3) and (4) until the process converges. Early trials indicated that the most efficient process was obtained with $\beta = 0$ and no iterations.

For the translational degrees of freedom equations (5) may be used directly.

For the rotational components equation (5a) cannot be used directly as the rotation terms (analogous to u_o or u_1) are described by unit vectors $\vec{b}_i$, where i = 1,2,3 whose rates are not equivalent to the angular velocities (analogous to v_o or v_1) and acceleration (analogous to a_o or a_1). A typical unit vector, $\vec{b}_3$, of equation (5a) with $\beta = 0$ takes the form

$$\vec{b}_3\Big|_{t+h} = \vec{b}_3 + \dot{\vec{b}}_3 h + \frac{1}{2} h^2 \ddot{\vec{b}}_3 \tag{6}$$

where $\vec{b}_3\Big|_{t+h}$ and $\vec{b}_3$ represent current and its previous unit vectors, respectively. Expanding into scalar components in the previous $[\bar{x},\bar{y},\bar{z}]$ system, these relations become

$$\begin{aligned} b_{3\bar{x}}\Big|_{t+h} &= + \bar{\omega}_y\, h + \frac{h^2}{2}\,(\bar{\omega}_x \bar{\omega}_z + \alpha_y) \\ b_{3\bar{y}}\Big|_{t+h} &= - \bar{\omega}_x\, h + \frac{h^2}{2}\,(\bar{\omega}_y \bar{\omega}_z + \alpha_x) \end{aligned} \tag{7}$$

with the third component, $b_{3\bar{z}}$, obtained from the condition that $\vec{b}_3$ remains a unit vector. A similar process is applied to another of the unit vectors, say $\vec{b}_2$, and the third vector is obtained from the cross product relation, $\vec{b}_1 = \vec{b}_2 \times \vec{b}_3$. The components of the new unit vectors referred to the global coordinate system then constitute the current transformation matrix [B]

2.2.2 Implicit Time Integration – The implicit integration procedure requires the formation of the tangent stiffness matrix and the matrix inversion for the solution of incremental displacements at each time step. The choice of this procedure results in a program having considerably more computer storage. However this procedure is capable of carrying out dynamic analysis at substantially greater time steps than are admitted in the explicit version. This capability is to include the limiting case of quasi-static crush phenomena involving slowly varying loadings and structural response.

The implicit integration formulation employs an incremental form of the equations of motion. This method requires a linearized estimate of the internal forces $\{F^{I}_{i+1}\}$ at the end of the time step in terms of the incremental forces. This is provided by the tangential stiffness matrix $[K_T]$,

$$\{F^{I}_{i+1}\} = \{F^{I}_{i}\} + [K_T]\ \{\Delta u\} \tag{8}$$

where $\{\Delta u\}$ are changes in displacements or rotations from the i to the i + 1 state, i.e., $\{u_{i+1}\}$ to $\{u_i\}$.

The Newmark-Beta method with $\beta = 1/4$ is used for the time integration; this is the constant average acceleration method. These formulas are

$$\begin{aligned} u_{i+1} &= u_i + h\ v_i + \frac{1}{4} h^2\ (a_i + a_{i+1}) \\ v_{i+1} &= v_i + \frac{1}{2} h\ (a_i + a_{i+1}) \end{aligned} \tag{9}$$

from which

$$\begin{aligned} \Delta u &= v_i\, h + \frac{1}{2} a_i h^2 + \frac{1}{4} h^2\ a_{i+1} \\ v_{i+1} &= v_i + \frac{h}{2}\ (a_i + a_{i+1}) \\ a_{i+1} &= \frac{4}{h^2}(\Delta u) - a_i - \frac{4}{h}\ v_{i+1} \end{aligned} \tag{10}$$

where h is the time interval.

The above recurrence formulas are valid both for increments in translations and rotations, provided that the increments in the latter are small, so that

$$\bar{\omega}_{i+1} = \frac{2}{h}\ \Delta\bar{\theta}_i - \bar{\omega}_i \tag{11}$$

The units vectors $\vec{b}_i$ are then expressed in terms of $\Delta\bar{\theta}$. The equations of motion are now derived.

The equations of translational motion can be obtained by substituting equation (8) into equation (3),

$$[M][\ddot{u}] + \{F_i^I\} + [K_T]\{\Delta u\} = \{F_{i+1}^E\} \tag{12}$$

Premultiplying equation (10a) by [M] and substituting in equation (12)

$$[K_T + \frac{4}{h^2}M]\,\{\Delta u\} = \{F_{i+1}^E\} - \{F_i^I\} + [M][\frac{4}{h}v_i + 1]\{a_i\} \tag{13}$$

Let the effective stiffness matrix, K^{eff}, be defined as

$$[K^{eff}] = [K_T + \frac{4}{h^2}M] \tag{14}$$

and let the right hand side of equation (13) be the effective force matrix, F^{eff}. Thus equation (13) simply becomes

$$[K^{eff}]\{\Delta u\} = [F^{eff}] \tag{15}$$

Equation (15) is solved at every time step for $\{\Delta u\}$. Velocities $\{v\}$ and accelerations $\{a\}$ at the step are found from equations (10a) and (10c).

The development of the equations of rotational motion proceeds as follows. Replacing the angular accelerations $\{\alpha\}$ and internal moments $\bar{F}_{i+1}^I$ in equation (4) by equations (10c) and (8), respectively, results

$$[K_T + \frac{4}{h^2}I]\,\{\Delta\theta\} = \{F_{i+1}^E\} - \{F_i^I\} - \{F_\omega\} + [I]\{\frac{4}{h}\omega_i + \alpha_i\} \tag{16}$$

where [I] are the mass moments of inertia matrix, see equation (4), $\{F_\omega\}$ was defined by equation (4) and ω_i and α_i are angular velocities (similar to v_i) and accelerations (similar to a_i), respectively. The form of the equations of rotational motion are similar to equation (13) for the translations. If the effective stiffness matrix, K^{eff}, is defined by the first term on the left hand side and the effective force matrix, F^{eff}, by the right hand side the equation is reduce to

$$[K^{eff}]\{\Delta\theta\} = \{F^{eff}\} \tag{17}$$

from which the change of rotation $\Delta\bar{\theta}$ can be determined. The current values of angular velocities and acceleration can be found from equation (11) and equation (10c), respectively.

The counterparts of equation (7), the updated unit vectors for the implicit procedure are

$$b_{3\bar{x}} = \left(\frac{h}{2}\,\bar{\omega}_z\Delta\bar{\theta}_x + \Delta\bar{\theta}_y\right)/\left(1 + \frac{h^2}{4}\,\bar{\omega}_z\omega_z\right)$$
$$b_{3\bar{y}} = \left(\frac{h}{2}\,\bar{\omega}_z\Delta\bar{\theta}_y + \Delta\bar{\theta}_x\right)/\left(1 + \frac{h^2}{4}\,\bar{\omega}_z\omega_z\right) \qquad (18)$$

The remaining components are then updated as described in the explicit formulation. In equation (8) it is understood that all angular velocities, $\bar{\omega}_x, \bar{\omega}_y$ and $\bar{\omega}_z$ are the current values obtained from equation (11).

Note that similarities between equations (15) or (17) and the equations of equilibrium for linear static problems. The effective stiffness matrix is simply inverted resulting in solutions for $\{\Delta u\}$ or $\{\Delta\theta\}$. In linear transient problems, this inversion process need only take place once, since the stiffness matrix is not dependent on the current stress state of the elements. This is not the case for nonlinear dynamics problems. The stiffness matrices must be recomputed at selected intervals to account for the nonlinearities in material behavior.

The most significant difference between equations (3) or (4) and (15) or (17) is that the implicit formulation requires a stiffness matrix. While it is true that this is much more taxing computationally, the capability of specifying larger time steps far outweighs the increased computational cost. Also, all of the routines developed for the explicit time integration are necessary in the development of an implicit computer program.

The major computational effort in this implicit procedure is in the triangulation of equation (15) or (17). The triangulation requires NB^2 multiplications where N is the number of degrees of freedom and B is the semibandwidth defined by $B = \max(I - J)$ in which I and J are the node numbers. Hence it is advantageous to minimize the semibandwidth when numbering the nodes.

2.3 Element Formulation

The treatment of large displacements and rotations employs a decomposition of the element displacement field into a rigid body rotation and translation associated with a local coordinate system attached to and moving with the element, and a remaining displacement field which describes the deformation of the element relative to the current position of the element axes. This transformation, in effect, removes the average rigid body rotation of the element and allows the use of small or moderate deflection element formulations in the calculation of element forces. In this manner, extremely large rotations and deflections can be accommodated by the analysis with accuracy depending primarily on the size of the elements relative to the curvature of the structure. Although a formal convergence theorem is lacking, the decomposition does hold for infinitesimal regions and numerical studies show excellent agreement with classical solutions.

The computer program at present includes low order triangular plate elements, three-dimensional beam elements and spring elements. A triangular membrane-hinge line element is also available but is not presently compatible with the beam formulation. Hinges and sliding joints or a combination of both are also available in beam elements.

2.4 Material Properties

The computer program currently uses simple elastic-plastic stress strain laws, a uniaxial relation for beam and spring elements and a biaxial strain hardening Mises model for plates. Element forces and bending moments for given strain fields are calculated by piecewise linear numerical integration of the stresses at selected points in the cross section. Options of an explicit moment curvature relationship for plates and the strain rate effect are also provided.

The explicit moment curvature formulation enables the user to make direct calculation of internal forces and moments from given extensions and curvatures of the plate element, thus negating the need for integration through the thickness. A necessary requirement of the procedure is that it results in shorter computer running times although at the expense of a loss in accuracy. The development is based on the adoption of an approximate yield function which considers the interaction of all bending moments and membrane forces acting on a plate section in determining the yield stress state of the section.

To decouple the complex phenomena of the various effects of strain rate on the elastoplastic material behavior, it is assumed that only the yield stresses are strain rate dependent and the rate of strain hardening remains unchanged. Such assumptions are considered to be adequate in the material formulation.

3. ANALYSIS RESULTS

A substantial number of test and demonstration problems have been treated during the development of the analysis and computer program reported herein. For the most part the test problems dealt with relatively simple structural configurations having known results either from exact solutions or from solutions by WRECKER I, and were selected to isolate and test particular aspects of the formulation and program during development. In all cases the analyses were pursued until satisfactory comparison with previous results were obtained or at least to the point at which differences between results could clearly be reconciled on the grounds of differences in loadings, boundary conditions, or ambiguities in the results used for comparison purposes. Selected results of these test problems are summarized in Table 1.

TABLE 1. SUMMARY OF TEST PROBLEMS

Problem	Description	Element Type and Material Properties	Purpose
1	Cantilever beam with lateral load applied at the tip	Two-dimensional beam element; elastic and elastic-plastic materials	a. Verify small and large deflection, both static and dynamic solutions b. Compare the explicit and implicit solutions c. Study the effects of the time increment on the accuracy of the implicit solution
2	Elastic cantilever plate with suddenly applied tip load	Elastic plate element; elastic material	a. Validate the elastic plate stiffness matrix b. Compare the explicit and implicit solution for dynamic problems c. Study the effect of the time increment on the accuracy of the implicit solution
3	Elastic square plate - Levy's plate	Elastic plate element; elastic material	a. Verify large static deflection solution
4	Dynamic loading on circular dome	Three-dimensional plate bending elements; elastic properties	a. Verify plate formulation for three-dimensional snap-through problem
5	Elastic square plate - Levy's plate	Elastic plate element; elastic material	a. Verify large static deflection solution
6	Elastic cantilever plate under a slowly applied laterial tip load	Elastic plate element; elastic material	a. Verify the quasi-static solutions b. Study the effect of the frequency of updating the matrixes on the solution

TABLE 1. concluded

Problem	Description	Element Type and Material Properties	Purpose
7	Spring subject to a suddenly applied load	Spring element; elastic and elastic-plastic materials	a. Validate the spring element b. Compare explicit and implicit (with different time step) solutions
8	Elastic-plastic cantilever plate with suddently applied tip load	Elastic-plastic plate element; elastic and elastic-plastic materials	a. Validate the elastic-plastic plate stiffness matrix b. Compare explicit and implicit solutions
9	Fixed end beam with intermediate hinge	Hinge joint: elastic materials	a. Validate the formulation of hinge joints, static solution
10	Fixed end beam with intermediate sliding joint	Sliding joint; elastic material	a. Validate the formulation of sliding joints, static solution
11	Fixed end beam with intermediate sliding joint with suddenly applied lateral load	Hinge joint; elastic material	a. Compare explicit and implicit solution
12	Frame with a roller support on inclined plane	Boundary element; elastic material	a. Simulate a roller support with a spring element
13	Trilinear stress-strain relation		a. Validate the trilinear stress-strain relation in subroutines for uniaxial and biaxial stresses
14	Strain-rate effect		a. Validate the strain-rate effect in a subroutine
15	Explicit moment-curvature relationship		a. Validate the explicit moment-curvature relationship in a subroutine

4. COMPARATIVE MERITS OF EXPLICIT AND IMPLICIT SCHEME

The choice between the explicit or implicit integration procedures depends mostly on the nature of the loading, the material type, and the finite element mesh. For impulsive type loads, the stability limitations on the selection of a time step are not critical, since an accurate determination of the response requires a small time step. A small time step is also required for highly path-dependent materials, such as some elastoplastic materials. Therefore, for this particular class of problems, it is apparent that explicit schemes are much more suitable since they generally require fewer computations for a given step.

The comparative computational merits of each scheme depend on the scale of the problem. In nonlinear dynamics problems, the stiffness matrix, required in an implicit scheme, varies with time and therefore must be recomputed, assembled and inverted at selected time steps during the solution. This inversion takes NB^2 computations, where N is the number of degrees of freedom and B is the semibandwidth of the stiffness matrix. In explicit schemes, the number of computations is directly proportional to the number of nodes. Therefore, for very large systems, explicit schemes may be far more economical; implicit schemes are more suitable for small bandwidth meshes.

Although the computational advantages of an explicit procedure appear to outweigh those of an implicit formulation, the small time step required for stability purposes in the explicit scheme is a major detriment in performing automobile crash simulation. In WRECKER I the solution time step was governed by the axial wave speed of the smallest element in the mesh. This generally required a time step of approximately 1 μsec for accurate solutions. In an implicit procedure the selection of a time step is not governed by the frequency content of the finite element mesh. This is especially attractive in the analysis of sheet metal components of automotive structures where axial (or membrane) response is not of major interest. For linear elastic analyses an implicit procedure is unconditionally stable regardless of the size of the time step. Recent work by Belytschko and Schoeberle* has demonstrated that implicit formulations are also unconditionally stable for elastoplastic analyses, provided that a specified energy criterion is satisfied. This key feature of implicit schemes allows quasi-static type analyses since the selection of a large time step (e.g., 1 sec) would reduce the inertial effects inherent in the problem, thereby rendering static type results. This is of particular importance when trying to reproduce full-scale "static" crush tests analytically.

Based upon the foregoing discussion, it may be said that although implicit schemes are more taxing computationally than explicit procedures, the ability to specify considerably larger time steps far surpasses the additional computational expense and leads to a more efficient computer procedure. However, it is again reiterated that this is only true for the class of problems currently under investigation. For rapid type loading and wave propagation problems the explicit procedures are clearly superior.

* Belytschko, T., and Schoeberle, D.F., "On the Unconditional Stability of an Implicit Algorithm for Nonlinear Structural Dynamics:, ASME, J. Applied Mechanics, December 1975.

5. CONCLUSIONS AND RECOMMENDATIONS

Results of a research project undertaken by IITRI for the Department of Transportation have been presented. The program objective was to develop a finite element computer program for use in the dynamic analysis of vehicle structure, including sheet metal, in a crash environment. This research program involved the following major tasks

- Modify the existing WRECKER I computer program to include an implicit solution algorithm thereby minimizing computer running time.
- Make the developed computer program more compatible with the user community by providing greater flexibility in modeling structures and preparation of necessary input data, and improvement of output data.
- Provide this analysis in a form compatible with other modules of a unified vehicle crash simulation system currently under development by NHTSA.
- Implement special beam end conditions into the developed computer program, thereby ball joints, pinned joints and rollers are available for the beam elements.
- Include calculations of the internal and external work as well as the kinetic energy, mass, and center of gravity.
- Overlay the developed computer program to minimize core memory requirements and to avoid overloading the EXEC8 operating system of the UNIVAC 1108.
- Facilitate the user in specifying general initial conditions and arbitrary loading functions by providing input in tabular forms (forces, displacements, and velocities as functions of time).
- Incorporate an elastoplastic spring element into the developed program for modeling the automotive suspension systems as well as special boundary conditions, such as rollers on inclined planes, for the structural system.
- Provide, as an option, approximate explicit moment-curvature relationships to be employed in simulating the nonlinear response of the plate element as a means of improving computational efficiency.

Make the trilinear stress-strain relationship and the strain rate effects optional to the user.

A substantial number of test and demonstration problems, both static and dynamic, were analyzed with the explicit or implicit procedures of this computer program.

5.1 Conclusions

Based on the work carried out in this research program, there are three principal conclusions:

(1) The analysis formulation, while somewhat novel, is a formally sound procedure and a proper and useful approximation technique for the dynamic analysis of beam and plate structures involving large deflections and rotations.

(2) The computer program developed is a correct rendering of this analysis technique and provided accurate results with reasonable efficiency in comparison to available known solutions.

(3) Finally, and most important, the computer program is readily applicable to realistic vehicle structures and provides credible results for actual crash events as demonstrated by the simulations on the end-on barrier tests in the WRECKER I program.

5.2 Recommendations

The analysis and computer program resulting from this research is a substantial improvement over previous attempts at a finite element analysis of vehicle structures, and is a potentially valuable addition to the tools available for the study of vehicle crashworthiness. Although many possibilities suggest themselves in terms of further development and refinement in the analysis procedure and computer program (note some below) the most important and potentially fruitful activity lies in the immediate and continued application of the analysis to actual vehicles and crash events. Such activity will bring the analysis to real vehicle related problems more quickly and will serve as a guide to its further development and application. Thus our primary recommendation is that provision be made for the continued and extensive use of the program in connection with actual vehicle behavior.

Program Applications:

Additional simulation of controlled crash events along the lines of those attempted in this work but extended to include machinery and drive train components and side-on impacts.

- Use of the program in connection with studies of alternate vehicle designs, including potential structural modifications and configuration changes.
- Use of the program to generate structural crush data for subsequent incorporation in the more simple lumped parameter or hybrid class of programs.
- Comparative studies of the deformation and crash-worthiness of existing vehicle designs.

Program Development:

- Development of a coupling procedure in which both explicit and implicit integration may be used within the same structure as a means of improving computational efficiency and accuracy.
- Incorporation of elements with composite material properties.
- Development of higher order and special hybrid element formulations possibly to include tearing or wrinkling within the element.
- Incorporation of larger strain formulations to simulate the sheet metal die stamping process.